FEM modeling of compressional systems: boundary flux and rheological effects on convergent orogens.

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Convergent plate boundaries or continental margins are characterized as zones of intense contractional deformation, regional metamorphism, and deep exhumation of crustal rocks. Modeling the mechanical and thermal processes that account for these characteristics presents a set of specific challenges. Foremost among these are problems of large strain associated with accretion of crustal material into a convergent orogen. Crustal accretion occurs at a variety of scales from offscraping of sediments to form an accretionary complex in oceanic subduction settings to accretion of large sections of continental crust as is occurring in the modern Himalaya. The transverse horizontal scales are similarly diverse with convergent orogenic plate boundaries varying from 100 kms to several thousand kms in width. At all scales, the accretionary process results in intense crustal deformation over a wide range of pressure and temperature conditions.

Numerical methods used to address lithosphere-scale deformation problems are generally based on finite element methods (FEM) incorporating a variety of mechanical parameterizations and constitutive relationships. One popular technique treats the lithosphere as a thin viscous sheet thereby neglecting vertical variations in strain rate and parameterizing lithospheric strength by a vertically averaged quantity. Another formulation, borrowed directly from mechanical engineering applications casts lithospheric deformation in terms of an elastic-plastic or a Maxwell visco-elastic material. This technique emphasizes the static, elastic stresses and strains in a Lagrangian (material) reference frame with plastic and viscous strains applied as a perturbation to the elastic solution. A third alternative, which I will describe and apply here, casts the problem as a non-linear viscous flow problem. With the appropriate choice of a strain rate-dependent viscosity a range of rheological behavior can be simulated including plastic materials.

Since the equations of motion for viscous materials are typically cast in an Eulerian (spatial) reference frame, this method has a solution mesh fixed in space rather than fixed to the material, thereby avoiding the numerical problems inherent to large deformation. The spatial mesh must be redefined to track any moving boundaries such as the free upper surface, but this is a straightforward process and does not lead to large mesh distortion. An Eulerian formulation also includes mass fluxes as a natural boundary condition, which permits easy incorporation of surface fluxes as occur with erosion and sedimentation. The disadvantages of the method are that elastic stresses and strains are neglected and if material properties are history dependant, the material paths must be tracked by a second, Lagrangian grid. The latter problem is time consuming, but tractable. The former has no easy solution although one important elastic process can be dealt with by compensating any crustal thickness changes using an analytical solution for flexure of an elastic plate.

I will illustrate applications of this numerical model with three examples. The first example addresses an outstanding structural problem, the observation of horizontal extension within convergent orogens. The second case is a study of the formation of orogenic plateaus by crustal thickening. Third, I present a study of the role of erosion in deep exhumation in convergent orogens and active margins. Each of these examples is discussed in depth below.

Horizontal extension in convergent orogens. Although convergent orogens exhibit dominantly contractional structures, there are many well-documented examples of structures which produce horizontal extension and formed within convergent orogens coeval with overall contraction. There are many potential explanations for these structures. In some cases local extension occurs as a result of geometric constraints imposed by structural heterogeneity, e.g. lateral ramps, or plate boundary geometry. Extension can also occur in a direction perpendicular to convergence if crustal thickening leads to lateral gravitational flow. However, in some cases extension appears to be in the direction of convergence and not related to specific structures. The modern collisional belt in Taiwan is a potential example of this process. The extension has been attributed to gravitationally-driven collapse of crustal thickness. In this model, crustal thickness increases by contraction until the gravitational forces exceed the 'strength' of the crust at which point the crust extends, thereby decreasing topography and thickness. However, this process is highly dependent on crustal rheology. A scalar 'strength' parameter for the crust implies a plastic rheology. In contrast, if the crust is characterized by a depth-dependent yield stress such as a Mohr-Coulomb criterion, there is no reason to expect additional topography to generate extensional failure. If the crust deforms as a viscous fluid, there is no stress threshold for failure and we would expect extensional strains to be always present.

This rheological dependence is demonstrated by a series of numerical models. A convergent orogen is parameterized by two-dimensional, plane strain deformation driven by far-field and basal displacements simulating substrate subduction with crustal accretion. A variety of crustal rheologies are investigated including linear viscous, power-law viscous, Coulomb plastic and combinations of the above. For all rheologies, subduction leads to the development of a doubly-vergent orogenic wedge that grows in a roughly self-similar manner. With a linear viscous rheology, the topographic crest of the orogen exhibits continuous horizontal extension. This extension is limited to the shallow crust to depths of the scale of the topography. Extensional strains can be on the order of 100 percent, but are limited by the process of asymmetric accretion, which tends to advect material out of the zone of extension before larger strains can accumulate. Power-law viscous materials also exhibit shallow extension, but at lower rates. In contrast, Coulomb plastic materials do not show any surface extension for a wide range of friction coefficients. The Coulomb models exhibit only contraction or no deformation across the upper surface and simply grow self-similarly with no limit to size. Models that include a Coulomb plastic 'lid' overlying a viscous lower crust also exhibit no extension. We can conclude that if this mechanism is important in orogens, it implies a bulk viscous behavior for the crust.

The formation of orogenic plateaus. At the largest scale, convergent orogens are characterized by topographic plateaus. The modern examples are the Altiplano of South America and the Tibetan Plateau in the India-Asia collision zone. In each of these cases the plateau is the result of structural thickening of the crust with attendant isostatic uplift. The Tibetan plateau is striking in its topographic uniformity with an average elevation of over 5 kms, even though it comprises a diverse collage of materials and structures in both the crust and upper mantle. The 'flatness' of these plateaus has been attributed to the strength of the lithosphere. The hypothesis is that convergent mountains grow by accretion until some limiting height is reached after which accretion is accommodated by outward growth of the orogenic plateau. This leads to some interesting questions regarding the mechanics of this process: (1) If the strength of the lower crust is limiting the elevation of the plateau, is the mantle completely decoupled from the crust? (2) What are the implications for lower crustal strength if plateau height is limited to 5 kms? (3) What is the space and time distribution of crustal thickening and elevation of an orogenic plateau? (4) How does a transient thermal state effect the strength of a crustal plateau, particularly with crustal heat production?

These questions are addressed by a series of numerical models of orogenesis and crustal plateau formation. Boundary conditions again simulate a process of subduction with crustal accretion and the two-dimensional plastic and viscous deformational response of the crust is modeled through the FE method described above. Applied total convergence is large enough to create an orogenic plateau. The formation of a plateau is a natural consequence of a temperature-dependent rheology. In the initial stages of crustal accretion, crustal thickening occurs to create an orogenic wedge consistent with its initial strength characteristics. This wedge grows self-similarly as new material is accreted. As the base of the wedge increases in depth and hence temperature, it becomes progressively weaker reaching a state in which no relief can be maintained over geologic timescales. This defines the maximum sustainable crustal thickness and the elevation of the plateau. Subsequent crustal accretion is accommodated by outward growth of this plateau. The time required to reach maximum elevation, the magnitude of this elevation and the deformation patterns implicit in its formation are a function of the convergence rate, the accreted crustal thickness, the effective viscosity of the lower crust, the thermal diffusivity and the internal heat production rate. These parameters scale through a set of characteristic numbers including a Peclet number and an Argand number. Results are shown for a range of these characteristic numbers and variations on the boundary conditions. Results depend only weakly on the boundary conditions because of the near complete decoupling between the crust and the mantle. Timescales of evolution depend primarily on the Peclet number which is an expression of the relative rates of crustal heating and crustal accretion. The equilibrium elevation of a plateau depends on the lower crustal viscosity expressed through the Argand number. The 5 km elevation of the Tibetan plateau implies a lower crustal viscosity of about 1019 Pa.s. The Altiplano plateau is complicated by the fact that it has formed by shortening of the overriding plate rather than crustal accretion, but the same thermal and mechanical processes apply once boundary conditions are modified to reflect this different formation mechanism.

Erosion and deep exhumation of convergent orogens. The exposure of deeper structural levels of an orogenic belt provides important information on internal kinematics and deformation as well as direct evidence for exhumational processes such as horizontal extension with vertical thinning and mass removal by surficial erosion. Extension was discussed in the first example; erosion is discussed here. Mass fluxes are easily dealt with numerically by working in an Eulerian reference frame. Since the numerical mesh is fixed in space, the normal component of nodal velocity represents a mass flux in or out of an element. If the boundary representing the free surface is moving, as occurs other than in the case of topographic steady state, it also has an associated velocity. Erosion is represented by an additional normal component to this surface velocity.

The effects of erosion on the evolution of a crustal orogen are illustrated by a series of models combining the tectonic model described above with an erosion model. The tectonic model is driven by subduction/accretion boundary conditions and includes plastic and temperature-dependent viscous deformational mechanisms. The erosion model is a one-dimensional fluvial incision model that erodes the upper surface at a rate proportional to surface slope and river discharge. River discharge is calculated by integrating a precipitation function downstream from the model-predicted water divide. Eroded mass is used to fill closed basins with the remainder allowed to leave the system.

The primary effect of erosion is to retard the rate of growth of an orogen. The positive feedback provided by the erosional dependence on slope and drainage area leads to a decreasing rate of growth and, eventually, a steady state of topography. Even with topographic steady state there is a continuous flux of material through the crust from point of accretion to the point of surface erosion. In a subduction system, there is a polarity to the kinematic paths of material points as all material is accreted from the subducting plate. This leads to a distinctive surface pattern of exhumation in which the deepest exhumation is exhibited by rocks on the upper plate on the side of the orogen opposite to subduction. This pattern of exhumation may be shifted or distorted relative to the topography, but remains a characteristic of the system. The exhumation pattern can also reach a steady state although the time to do so is much longer than that required to reach topographic steady state.

Geologic examples of modern orogens exhibiting high erosion rates, potentially steady state topography and asymmetric exhumation patterns include Taiwan, the Southern Alps of New Zealand, and the Olympic Mountains of Washington State. The comparison between the Southern Alps and the Olympic Mountains is particularly valuable as this contrasts orographic climate effects relative to the polarity of subduction. This comparison demonstrates that, as predicted by the models, exhumational and topographic asymmetry is maintained, but the patterns of exposure are distorted and shifted by the climatic effects.

References:

For background reading directly related to this talk, see the following and references within:

Willett, S.D., Orography and Orogeny: The Effects of Erosion on the Structure of Mountain Belts, J. Geophys. Res., Dec. issue, 1999.

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